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PROBABILITY MODELING AND ESTIMATION OF RISK MEASURES FOR FIRE LOSS SEVERITY IN PAKISTAN: AN APPLICATION OF EXTREME VALUE THEORY

Abstract. *Extreme events are increasing in the insurance and financial markets, causing large losses and ultimately huge insurance claims. Commercial fire loss severity has the largest value among the major insurance claims. The goal of our study is modeling the commercial fire loss severity and estimating the risk of extreme fire losses by using Extreme Value Theory (EVT). In the present study, we utilize the EVT (point over threshold modeling) for modeling the tail of fire loss data. We find that the Generalized Pareto distribution (GPD) gives more satisfactory fit to commercial fire loss data as compared to other parametric distributions including exponential, Pareto, gamma, logistic and generalized extreme value (GEV) distribution. In the empirical study, we determine the peaks over threshold of the GPD with the help of Mean Excess plots and Hill plots. We also estimate the risk measures like value at risk (VaR) and expected shortfall (ES). These estimates are helpful for pricing and risk management of non-insurance companies for their policy implications.*

Keywords: *fire loss, extreme value distribution, risk measures, value at risk, expected shortfall.*
JEL Classification: O30

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1. Introduction

In non-life insurance sector, only a few claims made by a portfolio often make majority of the compensations paid by the company. Among the major insurance claims, commercial fire insurance has the largest value. Therefore, modeling the tail behavior of fire loss severity is of great concern for pricing and risk management (Lee, 2012). Risk management is a process of thinking and identifying the all possible risks and problems before they happen and making decisions to control the risk or minimize its impact. It is a great challenge for a risk manager to implement the models for risk management and measure the consequences of damaging events.

Extreme value theory (EVT) is a statistical methodology which offers a framework for modeling the fire loss severities and their associated tail probabilities. An important application of the extreme value theory was introduced by the Longin(1996). He used extreme value theory to investigate the tail behavior of the extremes in the US stock market. He fitted generalized extreme value distribution to the data and concluded that the S&P 500 daily return data can also be characterized by the Frechet distribution which is a special case of generalized extreme value distribution. Beirlent (1992) used extreme value theory to model large claims. Zajdenweber (1996) modeled the claims of French insurance union data by using extreme value theory. Rootzen and Tajvidi (2000) used extreme value theory for fitting of wind storm losses. Hogg and Klugman (1984) fitted a truncated Pareto distribution to the loss data. Superiority of GPD could be viewed from the studies like, Resnik (1997), McNeil and Saladin (1997), McNeil (1999), Cebrián et al. (2003), Jondeau and Rockinger (2003), Beirlant et al. (2004), Lee and Fung (2010), Singh et al. (2011) and Wo-chiang lee (2012), in which they suggested that it is better to use Generalized Pareto distribution to extreme fire loss data. Extreme value theory is also used to estimate the risk measures like Value at Risk (VaR) and Expected Shortfall (ES). The risk managers can use the risk measures to assess the risk and make sure that their financial institute will survive after an extreme damaging event by setting the margin requirements. Cerovic (2014) showed that the VaR calculated by EVT is better than the econometric evaluations. Fernandez (2003) analyzed different ways of computing the VaR for different stock markets and concluded that the VaR based on EVT is the best one. Similarly, the following studies also advocate the use of EVT for the calculations of above risk measures such as Walls and Zhang (2005), Cheong vee et al. (2014) and Uppal (2013).

The next of the paper is organized as follow. Section 2 represents the methods and material, section 3 represents results and discussion, last section gives the concluding remarks and recommendation for further aspects of the study.

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2. Methods and Material

2.1. Extreme Value Theory (EVT)

First of all, the application of extreme value theory (EVT) was introduced by Gumble (1958). He developed the procedures for statistical estimation and presented different applications of extreme value theory in engineering and science. Now a days, EVT is also frequently used in financial modelling, risk management, insurance, telecommunication, meteorology and hydrology. In financial sector, EVT is used to identify and quantify the rare market and credit risks. EVT provides the basis of the procedures which are used in non-normal (Non-Gussian) market situation. It helps the risk manager to provide the models for risk management that are applicable in rare and damaging events. Extreme value theory is applicable for modelling the financial distributions like distribution of insurance claims, distribution of credit loss, distribution of returns and distribution of profit and loss. Extreme value theory provides a framework for modelling the rare or extreme events such as large insurance claims and market crashes by using statistical laws.

In extreme value theory, two basic methods are widely used. The oldest method is block maxima (BM) method, which was introduced by Gumble in 1958. In this method time series data is divided in non-overlapping groups or subsamples and then largest observation (maxima) is collected from each subsample. Block maxima follow Gumbel, Frechet or Weibull. This method involves two major drawbacks. Firstly, the largest observation is collected from the subsample or group so it drops various high extreme observations and retains some lower observation. Secondly, precision of the estimators is reduced because subsamples are used in this method.The modern group of methods is peaks over threshold (POT) method. The POT method was developed by the Pickands in 1975. This method is preferable when there is no large data or limiting data for extreme value analysis. In this method, all the high observations that exceed a high threshold are modelled. It is the most useful method in modelling the extreme events. The advantage of POT method over the block maxima method is that it uses all extreme observations present in the data. The probability distribution of the observations exceeding the chosen threshold approximately follows GPD (see Pickands 1975). It means, if enough data are available above threshold then we can use GPD as primary tool for modelling the tail of loss distribution. In the present study, we investigate the distribution of fire loss severity using POT method.

Let the random variable "x"has cumulative distribution function $F(x)$and let the threshold value is denoted by "u" which lies at the right tail of the distribution. Threshold value is said to be high if it lies close to the right end point. The probability that a random variable "x" lies between "u" and "$u + v$" is $F(u + v) - F(u)$ such that $v > 0$. The probability will be $(1 - F(u))$ if values of "x" being greater than the

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threshold value "u". $F_u(v)$ can be defined as the conditional probability that random variable lies between "u" and "u + v" given that $x > u$.

$$F_u(v) = \text{Prob}[x - u \leq v/x > u]$$

$$= \frac{F(v + v) - F(u)}{1 - F(u)} \quad (2.1)$$

The distribution of exceedances shows the probability of an event that exceeds threshold value "u" by a non zero value. The most critical problem in applying POT method is the selection of an appropriate threshold from where the tail begins of a distribution. By choosing a low value of threshold some of the observations from the center of the distribution are also included in the sample and the variance of the tail index becomes smaller (more precise) but biased. On the other hand, a high value of threshold results in a few exceedances that reduces the bias but it gives a large estimate of variance and the estimator becomes less precise. The most widely used graphical methods are Hill plot and Mean Excess plot used to choose an appropriate threshold.

Let $x_1 > x_2 > \cdots . > x_n$ be the order statistic of a random variable. The Hill estimator of the tail index based on (k+1) order statistic is gives as:

$$H_{k,n} = \frac{1}{k}\sum_{i=1}^{k}\{\ln x_{i,n} - \ln x_{k+1,n}\} \quad (2.2)$$

Where (k+1) are the number of upper order statistic and "n" is the sample size used in the estimation. Hill plot is used to choose the shape parameter estimate and threshold value of a data set. It is obtained that such estimates of the shape parameter are plotted as the function of the threshold value or k upper order statistics. The hill plot gives the maximum likelihood estimates of the shape parameter at different threshold points. The threshold value should be chosen from the tail region where the estimates of the shape parameter are approximately constant or where the graph looks like stable.

Davidson and Smith (1990) introduced the Mean Excess plot which is the plot of mean excess function (MEF). The MEF is simply the conditional mean of the excesses over the threshold value u and can be defines as:

$$(u) = \frac{1}{n_u}\sum_{i=1}^{n_u}(x_i - u) \quad (2.3)$$

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Where "u" is the threshold value and $n_u$denotes the total number of values which exceed the values resulting increment in threshold value. After threshold determination, the conditional distribution $F_u(v)$ defined in Equation (2.1). Convergence to GPD which usually expressed as three parameter distribution. We can determine the limit $F_u(v) \approx G_u(v)$ as $u \to \infty$ (Balkema (1974) and Pickand (1975)). The distribution function of the three parameter GPD can be expressed as:

$$G_u(v) = 1 - \left(1 + \varepsilon \frac{v}{\gamma}\right)^{-\frac{1}{\varepsilon}} \quad \text{if} \quad \varepsilon \neq 0 \tag{2.4}$$

Where "γ" is the scale parameter and "ε" is the shape parameter indicates the heaviness of the tail. The value of "ε"increases if the tail of the distribution becomes heavier (longer tailed).The probability that random variable exceeds the threshold value is $1 - F(u)$ and the probability that $x > u + v$ given that $x > u$ is$1 - G_u(v)$, so the unconditional probability that$X > u + v$ can be obtained as:

$$F(x > u + v) = [1 - F(u)] \cdot [1 - G(v)]$$

$[1 - F(u)]$ can be estimated by the empirical estimator$\left(\frac{k}{n}\right)$(Lee (2012)), where n is the total number of observations and k is the number of observations exceeding from the threshold value u. Hence, the unconditional probability that the $x > u + v$ is:

$$\frac{n_u}{n}[1 - G(v)] = \frac{k}{n}\left[1 - \left\{1 - \left(1 + \varepsilon \frac{v}{\gamma}\right)^{-\frac{1}{\varepsilon}}\right\}\right]$$
$$= \frac{k}{n}\left[\left(1 + \varepsilon \frac{v}{\gamma}\right)\right]^{-\frac{1}{\varepsilon}}$$

So the tail estimator for the cumulative distribution function can be defined as:

$$F(X) = 1 - \frac{k}{n}\left[\left(1 + \varepsilon \frac{v}{\gamma}\right)\right]^{-\frac{1}{\varepsilon}} \tag{2.5}$$

2.2. Value at risk and expected shortfall

We can calculate the VaR of GPD, infact, VaR is just like the quantile of a distribution corresponding to certain probability level "q". By the definition of VaR (for GPD):

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$$F(\mathrm{VaR}) = q$$

$$q = 1 - \frac{k}{n}\left[1 + \hat{\varepsilon}\frac{\mathrm{VaR} - u}{\gamma}\right]^{-\frac{1}{\varepsilon}}$$

$$\frac{k}{n}\left[1 + \varepsilon\frac{\mathrm{VaR} - u}{\gamma}\right]^{-\frac{1}{\varepsilon}} = 1 - q$$

$$\left[1 + \varepsilon\frac{\mathrm{VaR} - u}{\gamma}\right]^{-\frac{1}{\varepsilon}} = \frac{n}{k}(1 - q)$$

$$1 + \varepsilon\frac{\mathrm{VaR} - u}{\gamma} = \left[\frac{n}{k}(1 - q)\right]^{-\varepsilon}$$

$$\varepsilon\frac{\mathrm{VaR} - u}{\gamma} = \left[\frac{n}{k}(1 - q)\right]^{-\varepsilon} - 1$$

$$\mathrm{VaR} = u + \frac{\gamma}{\varepsilon}\left[\left\{\frac{n}{k}(1 - q)\right\}^{-\varepsilon} - 1\right] \quad (2.6)$$

Expected shortfall (ES) is an average loss given that your loss is greater than VaR (conditional value at risk). Indeed, the VaR only tells that your loss greater than some value which we call VaR with probability "p". It is unable to tell us how much greater. It may be any value which is greater than VaR. While expected shortfall is useful and giving extra information in the form of average of all possible loss given that the loss is greater than VaR. Expected shortfall is usually used in the field of financial risk management to find the market risk factor of a portfolio. At p% level, it may be defined as expected returns of the portfolio in the worst p% cases. For example, ES (0.1) is the expectation of the worst 10 cases out of 100 cases (Lee, 2012).By definition ES is defined as for confidence level "q"

$$\mathrm{ES}_q = E(\text{loss given that loss} > \mathrm{VaR}_q).$$

After solving the above conditional expectation, we get the following form for GPD.

$$\mathrm{ES}_q = \mathrm{VaR}_q + \frac{\gamma + \varepsilon(\mathrm{VaR}_q - u)}{1 - \varepsilon}$$

$$\mathrm{ES}_q = \frac{\mathrm{VaR}_q(1 - \varepsilon) + \gamma + \varepsilon(\mathrm{VaR}_q - u)}{1 - \varepsilon}$$

$$\mathrm{ES}_q = \frac{\mathrm{VaR}_q - \varepsilon\mathrm{VaR}_q + \gamma + \varepsilon\mathrm{VaR}_q - \varepsilon u}{1 - \varepsilon}$$

$$\mathrm{ES}_q = \frac{\mathrm{VaR}_q + \gamma - \varepsilon u}{1 - \varepsilon}$$

$$\mathrm{ES}_q = \frac{\mathrm{VaR}_q}{1 - \varepsilon} + \frac{\gamma - \varepsilon u}{1 - \varepsilon} \quad (2.7)$$

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3. Results and Discussion

3.1. Data description

There are four hundred sixty five observations in our data set. The five years commercial fire loss data used in this study is collected from the regional office of Rescue 1122 Rawalpindi. Table 1 shows the number of loss events, percentage of the loss events including sum and percentage of loss amount.

Table 1. Frequency of commercial fire loss data

Range of Loss amount (Rs.)	Number of loss events	Percentage of loss events	Sum of loss amount (Rs.)	Percentage of loss amount
0 – 100000	204	43.78	8013300	1.84
100001—200000	53	11.37	8077250	1.85
200001—400000	63	13.52	18228700	4.18
400001—1000000	79	16.95	49128000	11.26
1000001—5000000	50	10.73	104962500	24.07
Over 5000000	17	3.65	247737000	56.80
Total	**465**	**100**	**436146750**	**100**

Figure1 is visually describe fire losses in scatter plot, which indicates right skewness due to extreme losses along with the value of Skewness coefficient in Table 2. Results of the Table 2 also indicates that the series are highly positive are right skewed. It implies that the distribution of the data is heavy tail on right side. Kurtosis coefficient value is also very high which make it different from mesokurtic shape (Bell shape). In the present case, it has high peak and decline rapidly with heavy right tail. Table 2 reports the summary statistic of the fire loss data.

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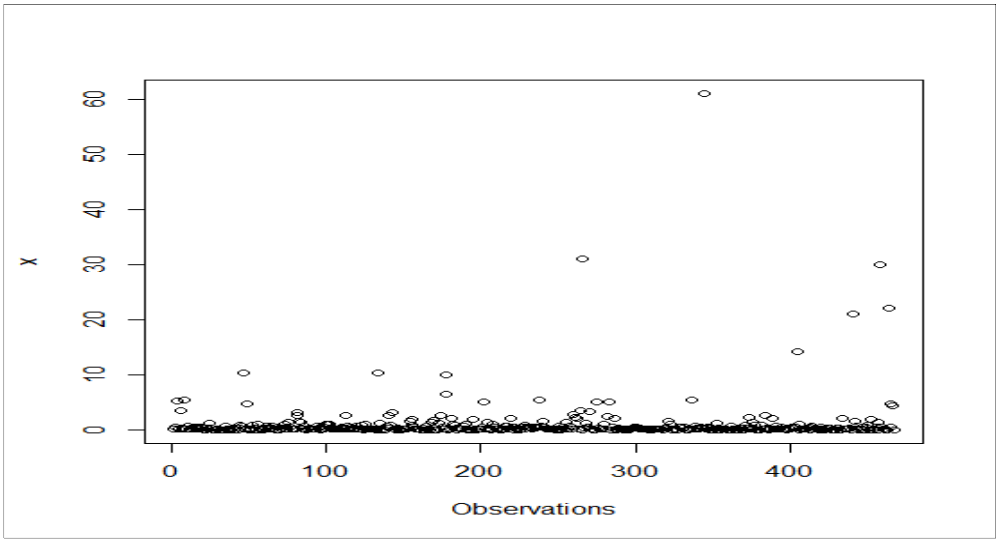

Figure 1. Scatter plot of the fire loss data

Table 2. Summary statistics of the fire loss data

N	Mean	S.D	C.V.	Skewness	Kurtosis	Min.	Max.
465	937950	3920700	4.1801	10.467	135.23	300	6.1×107

3.2. Selection of the probability model for commercial fire loss data

Practically it is not possible to consider all parametric probability distributions in a single study. However, one solution could be to consider a general class of distributions for fire loss data. Keeping the hope that these distributions would be flexible enough and conform the underlying data of fire loss severity in a reasonable way (Lee, 2012). We considered different parametric distributions like exponential, Pareto, gamma, logistic, generalized extreme value (GEV) and Generalized Pareto distribution (GPD) in this study. These distributions are selected because of their wide applications in the insurance and finance. The first step in our study is to explore the data. For exploratory data analysis we use the probability density plot and quantile-quantile (Q-Q) plot for each of the above distributions. These graphs help us to assess the goodness of fit of the parametric distributions at initial stage. In most of the financial problems the data series are fat tailed, so the Q-Q plot is more suitable for such series. The graph should be linear if parametric distribution fits the data well.Q-Q plot also help us to detect the outliers in the data set. Chi-Square (χ^2) test,

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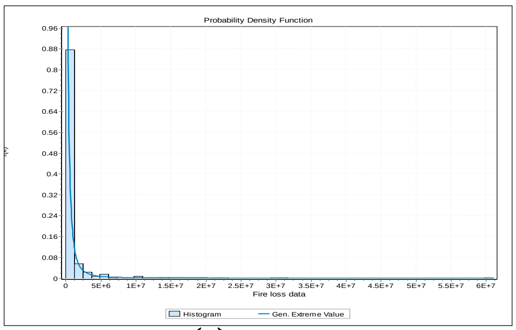
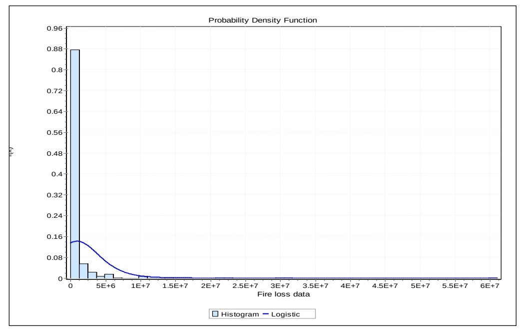
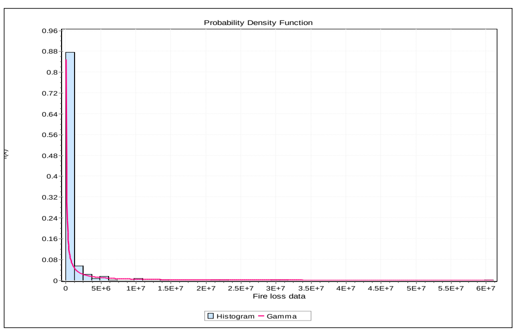
Kolmogorov-Simirnov (K-S) test and Anderson-Darling (A-D) test for the goodness of fit and also conducted to find which distribution is best fitted for fire loss data. For the verification of the results we calculated the RMSE and Bias for each distribution.



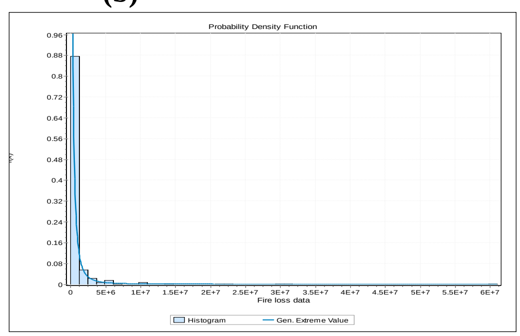
(a)



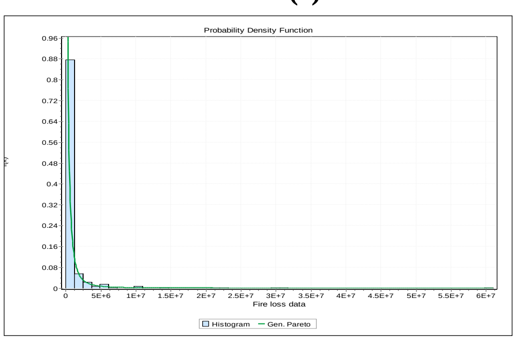
(b)



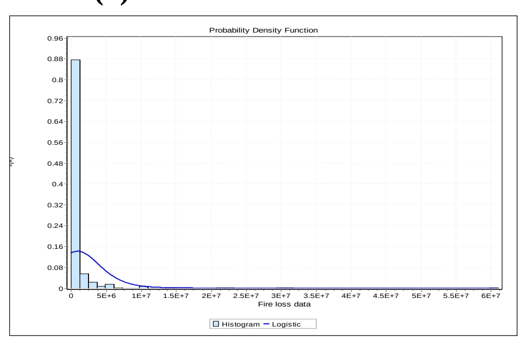
(c)



(d)



(e)



(f)

Figure 2(a-f). Probability density function (PDF) plots for fire loss data of different distributions

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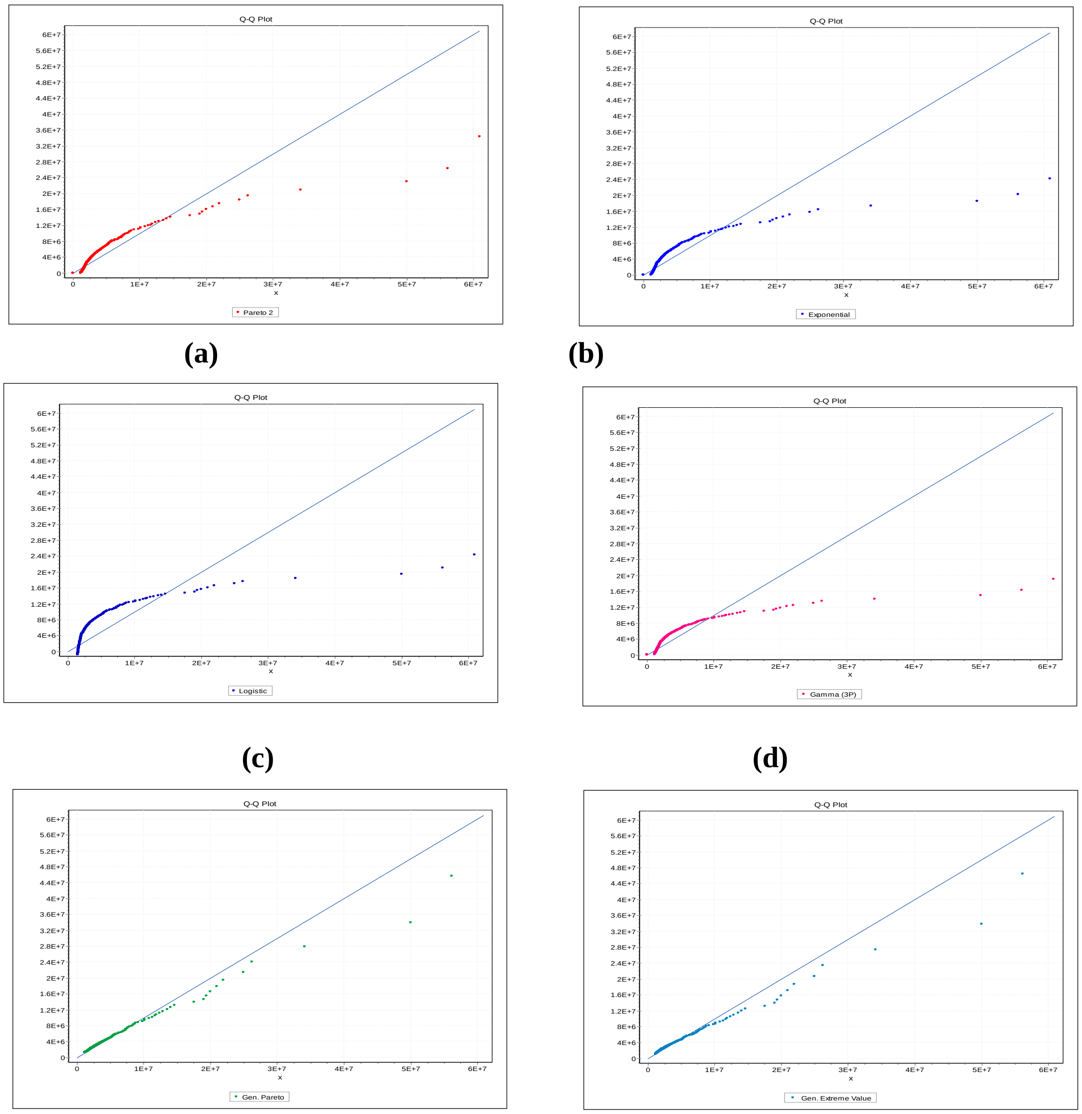

(a) **(b)**

(c) **(d)**

(e) **(f)**

Figure 3(a-f). Quantile-Quantile (Q-Q) plots for fire loss data of different distributions

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Figure 2(a-f) and Figure 3(a-f) show the poor fit of exponential, Pareto, gamma and logistic distribution, and better fit for GEV and GPD for fire loss data. Table 3 reports the parametric estimates of these fitted distributions.

Table 3. Parametric estimation for fitted distribution

Distribution	Location Parameter	Scale Parameter	Shape Parameter
Exponential	$\mu = 1.0662$	-	-
Logistic	$\mu = 2.1616 \times 10^6$	$\sigma = 9.3795 \times 10^5$	-
Pareto	-	$\sigma = 300$	$\varepsilon = 0.16275$
Gamma	-	$\sigma = 1.6389 \times 10^7$	$\varepsilon = 0.0572$
GEV	$\mu = 1.1243 \times 10^5$	$\sigma = 2.0063 \times 10^5$	$\varepsilon = 0.7836$
GPD	$\mu = 30462$	$\sigma = 2.3347 \times 10^5$	$\varepsilon = 0.7589$

Now we apply chi-square goodness of fit test, K-S test and A-D test to find out the most appropriate distribution for fitting the fire loss data. By using Easy Fit software (Version), we calculated the test statistic values (using MLE) of above three tests for goodness of fit and selected the distribution having lowest value of test statistic. Table 4 lists the test statistic values along their ranks by using different tests of goodness of fit for underlying distributions.

Table 4. Goodness fit results for different distributions

Distribution	**Chi-square (χ^2) test**		**K-S test**		**A-D test**	
	Statistic	**Rank**	**Statistic**	**Rank**	**Statistic**	**Rank**
Exponential	581.86	4	0.3723	4	199.61	5
Pareto	629.99	5	0.3575	3	98.959	3
Gamma	1068.8	6	0.6164	6	264.88	6
Logistic	229	3	0.3932	5	117.78	4
GEV	27.901	2	0.1242	2	8.0473	2
GPD	25.027	1	0.1180	1	6.0809	1

Table 4 shows that the GPD has least statistic value in all goodness of fit tests. So GPD is the best fitted model for fire loss data. For the purpose of verification, we use RMSE and bias measures. So we calculate the RMSE and bias for each distribution and GPD is giving the best fit as compared to others because it has smaller value of RMSE and bias which are defined as:

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$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \widehat{x_i})^2}{n-k}} \quad (3.1)$$

$$Bias = \frac{\sum_{i=1}^{n}(x_i - \widehat{x_i})}{n-k} \quad (3.2)$$

Where x_i are the observed values, $\widehat{x_i}$ are the fitted values, "n" is the sample size and "k" is number of estimated parameters. By using R software (Version:), we calculate RMSE and bias of selected distributions at different sample sizes (50, 100 and 200).

Table 5. RMSE and Bias of the fitted distributions

Distribution		**Sample size**		
		n=50	**n=100**	**n=200**
Exponential	**RMSE**	5880895	3621402	2890814
	Bias	891970.5	329035.3	15467.5
Pareto	**RMSE**	4024990	3127567.8	2032971
	Bias	3513588	258233.3	2403371
Gamma	**RMSE**	5071554	2744707	2445375
	Bias	378826.4	254677.5	2386626
Logistic	**RMSE**	8724311	6467891.5	4498762
	Bias	752278	634589.8	538251.1
GEV	**RMSE**	2464638	1624977	1456378
	Bias	260519.5	139697.8	106740.1
GPD	**RMSE**	2398323	1232139	1114895
	Bias	241078.97	123292.3	81305.3

Table 5 indicates that the GPD has lowest RMSE and bias at different sample sizes. So, in context of exploratory analysis, different goodness of fit tests and on the basis of RMSE and bias, we can say that the GPD is the best fitted model for the fire loss data. Threshold values and order statistics are taken on x-axis while on y-axis the values of shape parameter are shown in Figure 4. The threshold would be selected from the plot where we observe stability of the shape parameter. For more than one threshold, we can use mean excess plots. Mean excess plot also help us to depict the threshold. In Figure 5, the sample mean excesses are plotted against the threshold values. Threshold values (in millions) are taken on x-axis and mean excesses are taken on y-axis. The plot shows an upward slope which indicates the heavy tail of the sample data. In upward sloping, three segments can be seen. In first segment the value of threshold is almost 6.0×10^5 and in other two segments the threshold values are $1.01\times10^6$and 1.95×10^6.

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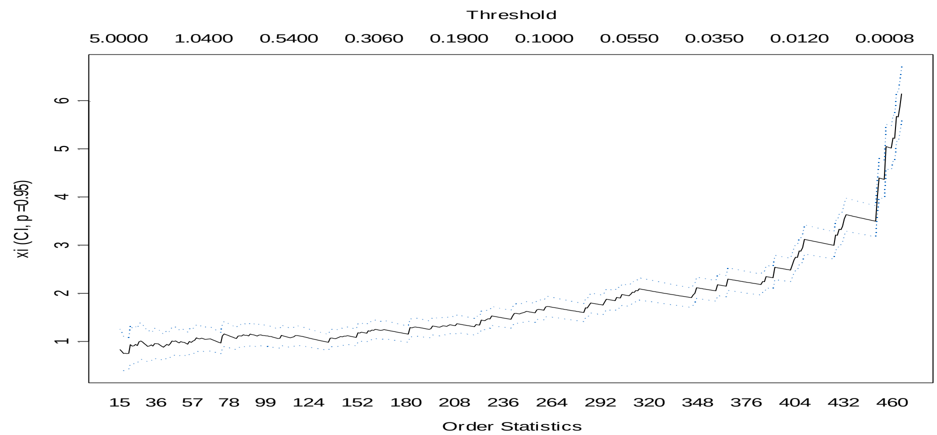

Figure 4. Hill plot for fire loss data

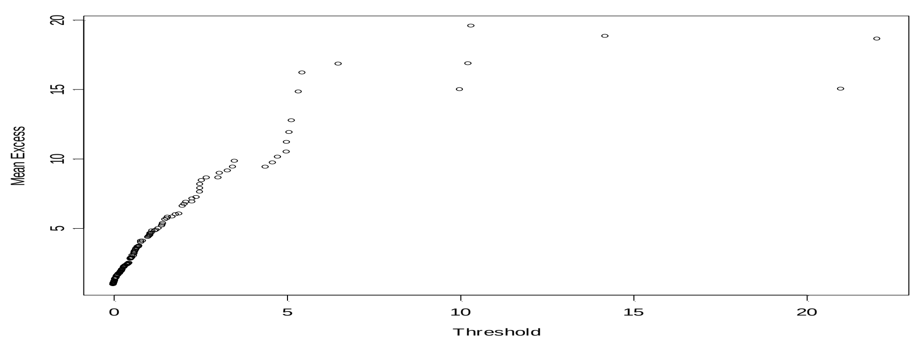

Figure 5. Mean excess for fire loss data

In Table 6, the value of "ε" is fairly stable around the threshold value $6.0\times10^5$and have 100 exceedances. Similarly in next two segments the two other threshold values are $1.01\times10^6$and 1.95×10^6 where the value of ε is stable.

In Figure 6. we plot the cumulative distribution function of estimated GPD model and fire loss data above the three threshold values. It also depicts that GPD model reasonably fit the fire loss data used in this study at three selected thresholds.

Now, we test the goodness of fit for GPD model at these threshold values by using Kolmogorov-Smirknove test and chi-square goodness of fit test.

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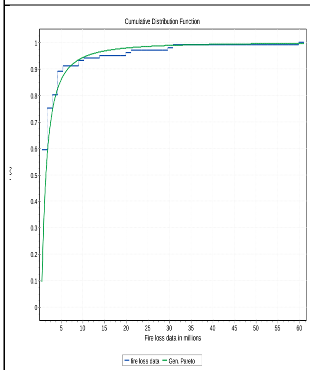
The null hypothesis regarding fire loss data follow generalized Pareto distribution. The results of goodness of fit for GPD over different threshold values are summarized in Table 7.

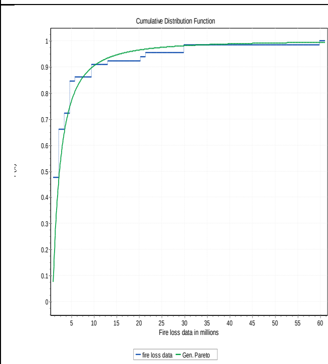
Table 6. Threshold values where the shape parameter of GPD is fairly

Threshold value 6.0×10^5			**Threshold value 1.01×10^6**			**Threshold value 1.95×10^6**		
ε	**No. of excee-dances**	**Threshold**	**ε**	**No. of excee-dances**	**Threshold**	**ε**	**No. of excee-dances**	**Threshold**
1.101	101	5.9×10^5	1.053	67	8.5×10^5	0.950	37	1.90×10^6
1.102	100	6.0×10^5	1.053	66	1.00×10^6	0.950	36	1.90×10^6
1.102	99	6.0×10^5	1.054	65	1.02×10^6	0.950	35	2.00×10^6
1.102	98	6.1×10^5	1.054	64	1.04×10^6	0.951	34	2.10×10^6

Threshold value = 6.0×10^5



Threshold value = 1.01×10^6



Threshold value = 1.95×10^6

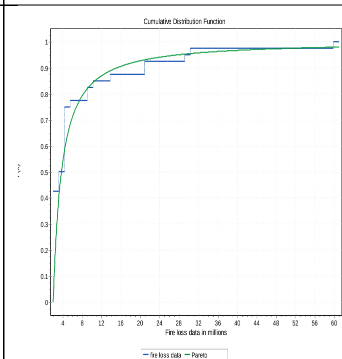


Figure 6:CDF plots of estimated GPD model and the fire loss data of above threshold

Table 7 reveals the fact that the both K-S test and chi-square test does not reject H_0. which means that fire loss data has a GPD distribution and it is good for model fitting.

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Table 7. Goodness of fit for GPD over threshold values

	No. of exceedances		
	100	**65**	**35**
Threshold value	***6.0×10^5***	***1.01×10^6***	***1.95×10^6***
K-S test (p-value)	**0.09532** (0.29856)	**0.07519** (0.82241)	**0.13625** (0.41098)
Chi-square test (p-value)	**2.2417** (0.89618)	**2.8009** (0.83339)	**4.619** (0.20192)

3.3. Calculation of risk measures

Table 8 reports the estimates of shape and scale parameters along with their standard error of GPD, number of exceedances and risk measures estimates at different confidence levels are also highlighted in Table 8. For example we observe that when the threshold is 6.0×10^5 then the number of exceedances is 100. The shape parameter is 0.7898 with a standard error 0.1630 (given in parentheses) which indicates a heavy tail. The value at Risk (VaR) and expected shortfall(ES) are calculated at different confidence levels. Using a confidence level of 95%, the value of VaR. i.e. 3.1019×10^6reveals that there is 5% probability that minimum loss would be equal to 3.1019×10^6 or greater (gain) or 95% confident that the maximum loss would be equal to 3.1019×10^6 or less. We are 5% confident that the amount of extreme loss would increase the amount 3.1019×10^6 i.e. VaR. This quantity is unable to tell further how much greater than this VaR amount. To know about this, we use another quantitative and synthetic measure known as ES. It tells us about the average loss given that the amount of loss is greater than VaR with certain probability level. It may be any value greater than *VaR*.ES is useful in the sense that it is giving extra information in the form of average of all possible loss given that the loss is greater than *VaR.* It is a good measure for risk management of portfolio. For example, ES (0.05) with confidence level of 95%, it may be defined as the expectation of the worst 5 cases out of 100 cases provided your loss is greater than VaR (0.05).

Table 8.Value at risk and expected shortfall

	No. of exceedances		
	100	***65***	***35***
Threshold value	***6.0×10^5***	***1.01×10^6***	***1.95×10^6***
Scaling parameter	***9.40×10^5***	***8.312×10^5***	***9.73×10^5***

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	(2.037×10^5)	***(2.281×10^5)***	***(3.427×10^5)***
Shape parameter	***0.7898,(0.1630)***	***1.3344,(0.3178)***	***1.6500(0.5687)***
VaR (95%)	3.1019×10^6	3.2260×10^6	3.1293×10^6
VaR (97%)	5.1619×10^6	5.2475×10^6	4.9804×10^6
VaR (99%)	1.4932×10^7	1.3967×10^7	1.4018×10^7
ES (95%)	5.5472×10^7	2.2046×10^7	1.0667×10^8
ES (97%)	8.9811×10^7	3.4023×10^7	1.7519×10^8
ES (99%)	2.5268×10^8	8.5682×10^7	5.0971×10^8

4. Conclusion

Fitting the tail of loss data has great concern in many applications of the finance and insurance. The estimates of the tail have great importance for the risk management. In our present study we determined the most appropriate distribution for modeling the extreme fire losses. We first executed the exploratory analysis by using probability density plot and quantile-quantile (Q-Q) plot of exponential, Pareto, gamma, logistic, generalized extreme value and generalized Pareto distribution. Both plots revealed a poor fit of exponential and logistic distribution while other distributions fit the data much better especially the GPD and GEV distribution. We applied the chi-square test, Kolmogorov-Smirknove test and Anderson Darling test for goodness of fit to the fire loss data. All the tests for goodness of fit support the GPD as the best fitted distribution. We also verify our results by computing root mean square error and bias for each distribution and found that the GPD has minimum root mean square error and bias. So, we concluded that the GPD is the appropriate distribution for the fore loss distribution. For modeling the tail of fire loss data, we used the peaks over threshold method and determined the optimal thresholds by Hill estimator and Mean Excess plot. We fitted the GPD over the thresholds and computed the value at risk and expected shortfall at different confidence levels over the high thresholds 6.0×105, 1.01×106 and 1.95×106.These estimates would be helpful for pricing and risk management of non-insurance companies for their policy implications. For example, risk managers can use the risk measures to assess the risk and make sure that their financial institute will survive after an extreme damaging eventby setting the margin requirements.

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